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Problem Set #3

(1)

(a)

(i) We know that the Average or Expected Value is given by the following:

$\overline{R} = E(R) = p_1R_1 + p_2R_2 + ... + p_nR_n$

Thus, the expect value of the rate of return on our client's portfolio is:

$\overline{R} = E(R) = 0.60 \times 15 + 0.40 \times 6 = 11.4\%$

(ii) The standard deviation is simply the square root of the variance.

Thus, $\sigma = \sqrt{\sum_{k=1}^{n} p_k (R_k - \overline{R})^2}$

However, we must note that we have a 'two stock' portfolio. With T-Bill having S.D. of 0.

Thus, our portfolio variance $= x_1^2\sigma_1^2 + 2x_1x_2p_{12}\sigma_1\sigma_2 + x_2^2\sigma_2^2$ and since $\sigma_2 = 0$
Our portfolio variance becomes: $x_1^2\sigma_1^2 = 0.60^2 30^2 = 324$

Thus: $\sigma = \sqrt{324} = 18$

(b) In order to find out the proportions of our client's overall portfolio, including the position in T-Bills we first need to find out what percentage do Stock A and Stock B take up.

$StockA = (.25)(.60) = 0.15$ Of the whole portfolio
$StockB = (.75)(.60) = 0.45$ Of the whole portfolio

Thus, the overall investment proportions of our client's portfolio are:

StockA=0.15
StockB=0.45
T-Bill=0.40

(c)
In order to find out what proportion should be invested in our fund and T-Bills,
In order to obtain a 14% return, we simply need to solve the following equation:

$$\overline{R} = E(R) = P \times 15 + (1 - P) \times 6 = 14\%$$

Solving this we get:

$$\overline{R} = E(R) = P \times 15 + (1 - P) \times 6 = 14\%$$
$$15P + 6 - 6P = 14$$
$$9P = 8$$
$$P = 8/9$$

Thus, our client should invest $8/9$ of her wealth into our Fund, and $1/9$ into the T-Bills

(d)

Here, we want to maximize the expected rate of return $(15x + 6(1 - x))$ subject to $\sigma = \sqrt{x^2 30^2} \leq 25$. It is evident that in order to maximize the expected rate of return we would need to invest as much as possible into our Fund. So in other words, in order to maximize our expected return on our portfolio, we simply need to solve the following equation :

$$\sqrt{x^2 30^2} = 25$$
$$x^2 30^2 = 25^2$$
$$x = \sqrt{\frac{25^2}{30^2}} = 5/6$$

So we need to invest $5/6$ into our Fund, and $1/6$ into T-Bills.

And the expected return of the portfolio will be:

$$1/6(6) + 5/6(15) = 13.5\%$$

(e)
She didn't become more or less risk averse. If we compare part (c) and part (d) our client accepted a lower rate of return (in part (d)) with lower risk. A risk averse

person only accepts higher or lower rate of return provided she or he is compensated with adequate rate of return. In this example our client's risk level moves in the same direction as her rate of return, thus we cannot tell weather or not she is more or less risk averse.

(2)

First of all, it would be helpful to write all of the information in 1 place. So we have:

Individual Investment break-down:

	GSB	HBS	QED
Townsend	200	1200	600
Axelrod	400	100	500
Plunkett	200	250	50

Expected return and Standard Deviation:

	Expected Return	Standard Deviation
GSB	0.20	0.40
HBS	0.16	0.30
QED	0.10	0.20

Corr(GSB,HBS)= -0.20
Corr(GSB,QED)= 0.20
Corr(HBS,QED)= 0.50

So in order to find out which portfolio is the "cleaverest" we first compute the expected return of each portfolio as well as standard deviation…

Townsend:

$$E(P) = 0.20 * (200/2000) + 0.16 * (1200/2000) + 0.10 * (600/2000) = 14.6\%$$

$$\sigma = \sqrt{X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2(X_1)(X_2)\sigma_{21} + 2(X_3)(X_1)\sigma_{31} + 2(X_2)(X_3)\sigma_{23}} =$$

$$= \sqrt{\begin{matrix}(200/2000)_1^2(0.40)_1^2 + (1200/2000)_2^2(0.30)_2^2 + (600/2000)_3^2(0.20)_3^2 + 2((200/2000)_1)(1200/2000_2)(-0.20)_{21} \\ +2(600/2000_3)((200/2000)_1)(0.20)_{31} + 2(1200/2000_2)(600/2000_3)(0.50)_{23}\end{matrix}} =$$

$$= 45.3\%$$

Axelrod:

$$E(P) = 0.20 * (400/1000) + 0.16 * (100/1000) + 0.10 * (500/1000) = 14.6\%$$

$$\sigma = \sqrt{X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2(X_1)(X_2)\sigma_{21} + 2(X_3)(X_1)\sigma_{31} + 2(X_2)(X_3)\sigma_{23}} =$$

$$= \sqrt{\begin{array}{l}(400/1000)_1^2(0.40)_1^2 + (100/1000)_2^2(0.30)_2^2 + (500/1000)_3^2(0.20)_3^2 + 2((400/1000)_1)(100/1000_2)(-0.20)_{21} \\ +2(500/1000_3)((400/1000)_1)(0.20)_{31} + 2(100/1000_2)(500/1000_3)(0.50)_{23}\end{array}} =$$

$$= 38.7\%$$

Plunkett

$$E(P) = 0.20 * (200/500) + 0.16 * (250/500) + 0.10 * (50/500) = 17\%$$

$$\sigma = \sqrt{X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2(X_1)(X_2)\sigma_{21} + 2(X_3)(X_1)\sigma_{31} + 2(X_2)(X_3)\sigma_{23}} =$$

$$= \sqrt{\begin{array}{l}(200/500)_1^2(0.40)_1^2 + (250/500)_2^2(0.30)_2^2 + (50/500)_3^2(0.20)_3^2 + 2((200/500)_1)(250/500_2)(-0.20)_{21} \\ +2(50/500_3)((200/500)_1)(0.20)_{31} + 2(250/500_2)(50/500_3)(0.50)_{23}\end{array}} =$$

$$= 14.1\%$$

Thus, the murderer is clearly Plunkett because his portfolio produces the greatest expected return with minimal risk!

(3)

(i) First we compute the expected return for EWeight index:

$$E[EWeight] = 0.5(10) + 0.5(15) = 12.5$$

And standard deviation:

$$\sigma = \sqrt{(0.5)^2(15)^2 + (0.5)^2(25)^2 + 2(0.5)(0.5)(0.60)} =$$
$$= 14.58$$

And now we compute the same for VWeight index:
(Since stock 1 has twice the market value of stock 2, we will hold twice as much stock1 than stock 2)

$$E[VWeight] = (2/3)(10) + (1/3)(15) = 11.66$$

And the standard deviation is:

$$\sigma = \sqrt{((2/3))^2(15)^2 + (1/3)^2(25)^2 + 2(1/3)(2/3)(0.60)} =$$
$$= 13.027$$

In my 401K I would choose VWeight and money market because EWeight will only give us 1 more % of return with 1.5 more units of risk.

Of course, if I was not risk averse, I would probably choose EWeight even though it posses greater risk.