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FINAL PROBLEM SET

PROBLEM SET No. 4

UNIVERSITY OF CALIFORNIA SANTA BARBARA
DEPARTMENT OF ECONOMICS

HOMEWORK NO. 4

Question:

You can produce output using either of two machines. A type y machine will last $y \in \{1,2\}$ years and costs K_y to purchase, with $K_1 = 1 < K_2 = 1.5$. Either machine can produce output per period q at cost q^2 . You are a price taker. The price follows a Markov chain defined on two states H and L . The transition probabilities are given by:

$$Pr(\tilde{p}_{t+1} = H = 3 | \tilde{p}_t = H = 3) = \pi_1 = 0.7$$

$$Pr(\tilde{p}_{t+1} = L = 1 | \tilde{p}_t = L = 1) = \pi_2 = 0.4$$

You can have only one machine per period, you can buy a machine only after observing the price of output, and you cannot resell a machine. The firm chooses output $q(t)$ and machine purchase $m(y,t)$ so as to maximize expected profits

$$E \sum_{t=1}^{\infty} \beta^t (p_t q_t - q_t^2 - m(1,t)K_1 - m(2,t)K_2)$$

where $\beta = 0.8 \in (0,1)$ is the discount factor and this maximization is subject to the constraints already mentioned. What should you do? Use a computer to solve this problem. What is the long-run distribution of machines in this industry?

Answer:

As the problem indicates we have two states, namely $X = \{H, L\}$

As was discussed in lecture, we first must determine the value or position at state zero. In other words we must determine:

$$V_0(L) = ?$$

$$V_0(H) = ?$$

By using the reward function described in the homework, we can build the following matrix which consists of values in the low and the high period having purchased machines 1 or 2. We will maximize our profits when by choosing the maximum value of corresponding machine purchase in each state

beta	0.8	N	v0high	v0low	v1high	v1low	r0low0	r0low2	r0high1	r0high2
kay1	1	0	0.00	0.00	0.00	0.00				
kay2	1.5	1	1.25	0.00	2.25	0.25	0.00	-1.25	1.25	0.75
hi	3	2	2.07	0.60	2.95	0.85	0.60	-0.09	1.95	2.07
lo	1	3	2.61	1.19	3.55	1.44	1.19	0.44	2.55	2.61
pie1	0.7	4	3.08	1.63	3.99	1.88	1.63	0.91	2.99	3.08
pie2	0.4	5	3.44	2.00	4.37	2.25	2.00	1.27	3.37	3.44
		6	3.74	2.29	4.66	2.54	2.29	1.57	3.66	3.74
		7	3.97	2.53	4.89	2.78	2.53	1.80	3.89	3.97
		8	4.16	2.71	5.08	2.96	2.71	1.99	4.08	4.16
		9	4.30	2.86	5.23	3.11	2.86	2.14	4.23	4.30
		10	4.43	2.98	5.35	3.23	2.98	2.26	4.35	4.43
		11	4.52	3.08	5.44	3.33	3.08	2.35	4.44	4.52
		12	4.60	3.15	5.52	3.40	3.15	2.43	4.52	4.60
		13	4.66	3.22	5.58	3.47	3.22	2.49	4.58	4.66
		14	4.71	3.27	5.63	3.52	3.27	2.54	4.63	4.71
		15	4.75	3.30	5.67	3.55	3.30	2.58	4.67	4.75
		16	4.78	3.34	5.70	3.59	3.34	2.61	4.70	4.78
		17	4.80	3.36	5.73	3.61	3.36	2.63	4.73	4.80
		18	4.82	3.38	5.75	3.63	3.38	2.65	4.75	4.82
		19	4.84	3.40	5.76	3.65	3.40	2.67	4.76	4.84
		20	4.85	3.41	5.78	3.66	3.41	2.68	4.78	4.85
		21	4.86	3.42	5.79	3.67	3.42	2.69	4.79	4.86
		22	4.87	3.43	5.79	3.68	3.43	2.70	4.79	4.87
		23	4.88	3.44	5.80	3.69	3.44	2.71	4.80	4.88
		24	4.88	3.44	5.81	3.69	3.44	2.71	4.81	4.88
		25	4.89	3.44	5.81	3.69	3.44	2.72	4.81	4.89
		26	4.89	3.45	5.81	3.70	3.45	2.72	4.81	4.89
		27	4.89	3.45	5.82	3.70	3.45	2.72	4.82	4.89
		28	4.90	3.45	5.82	3.70	3.45	2.73	4.82	4.90
		29	4.90	3.45	5.82	3.70	3.45	2.73	4.82	4.90
		30	4.90	3.46	5.82	3.71	3.46	2.73	4.82	4.90