

Problem Set #6

(I)

Ricardia has 200 workers, and 60 acres of good land (2workers=50bushels of wheat) and 40 acres of bad land (4 workers/acre = 60 bushels of wheat)

- (a) We know that we will hire as many workers as possible for the good land, because it will give us the most profit. We need 120 workers for the good land (60*2) which leaves 80 workers for the bad land. We know that for bad land: $q=(60)(1/4)$. In other words, 4 workers produce 60 bushels. Thus the marginal productivity of hiring one extra worker is:

$$MP = \frac{\partial q}{\partial l} = 60/4 = 15 \text{ bushels of wheat/worker}$$

- (b) The rent of the good land is revenue derived from it, minus the cost. So, $\text{Rent}(\text{good})=50 \text{ bushels/acre}-(15 \text{ bushels/acre})\cdot 2 = 20 \text{ bushels/acre}$

The rent of the bad land is computed in a similar manner:
 $\text{Rent}(\text{bad})=60 \text{ bushels/acre}-(15 \text{ bushels/acre})\cdot 4 = 0$

- (c) The marginal productivity of good land is simply: $MP = \frac{\partial q}{\partial l} = 50/2 = 25$ and for bad land it is 15 bushels/worker. It is also important to note that with unlimited amount of workers, this case is not binding.

(II)

Buyer's production function: $q = 728K - 2K^2$

Seller's production function: $K = X^{0.5}$

$$p_q = 6$$

$$p_x = 3$$

(a) *Monopoly*

We know that in a monopoly setting, there exists only one seller. Thus, we should compute the price and quantity of K by looking at the production function of the seller, and noting where $MR=MC$:

The cost and the revenue are equal by the following: (for buyer)

$$R = 4368K - 18K^2$$

$$MR = 4368 - 36K$$

$$C = Kp_k$$

$$MC = p_k$$

$$MC = MR$$

$$4368 - 36K = p_k$$

And for seller:

$$R = p_k K = 4368K - 36K^2$$

$$MR = 4368 - 72K$$

$$C = p_x X = 3K^2$$

$$MC = 6K$$

Thus

$$MC = MR$$

$$4368 - 72K = 6K$$

$$K = 56$$

$$p_k = 2352$$

(b) *Monopsony*

In monopsony there is one buyer and many sellers.

We know that in a monopsony setting: $MR=MC$

The profit for the seller is as follows:

$$p_k K - p_x X = p_k K - p_x K^2$$

$$\frac{\partial \pi}{\partial K} = p_k - 2p_x K$$

$$p_k = 6K$$

And for the buyer:

$$\begin{aligned}
p_q q - p_k K &= \text{profit} \\
\Rightarrow 6(728K - 3K^2) - 6K^2 \\
\Rightarrow 4368K - 24K^2 \\
\frac{\partial \pi}{\partial K} &= 4368 - 48K = 0
\end{aligned}$$

Thus we can compute:

$$\begin{aligned}
K &= 91 \\
p_k &= 546
\end{aligned}$$

(c) Competition

The seller and the buyer competing...

For the buyer we have:

$$\begin{aligned}
p_q q - p_k K &= \text{profit} \\
\Rightarrow 6(728K - 3K^2) - p_k K \\
\Rightarrow 4368K - 18K^2 - p_k K \\
\frac{\partial \pi}{\partial K} &= 4368 - 36K - p_k = 0 \\
\Rightarrow p_k &= 4368 - 36K
\end{aligned}$$

And for the seller:

$$\begin{aligned}
p_k K - p_x X &= \text{profit} \\
p_k K - 3K^2 &= \text{profit} \\
\frac{\partial \pi}{\partial K} &= p_k - 6K
\end{aligned}$$

So we can compute the price of K and the quantity sold:

$$\begin{aligned}
4368 - 36K &= 6K \\
\Rightarrow K &= 104 \\
\Rightarrow p_k &= 624
\end{aligned}$$