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Econ 205A
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Problem Set #4

[1]

(a)

Given:

$U = xy$

$x = 2$

$y = 1$

- We need to find how much she can spend on the transportation given the condition that she cannot be made worse off

Solution:

- We know that initially utility is: $U = 2 * 1 = 2$
  $2 = xy$
- Thus or
  $y = 2/x$
- We also want to minimize the sum of x and y (holding utility constant). Right now $x + y = 2 + 1 = 3$
- We want that number to be as small as possible. Thus our conditions are as follows:
  $x + y = z$
- $xy = 2$
- Where z is the minimal value.
  $2/x = z - x$
- $z = 2/x + x$
- Thus, our job now is to find where that function is minimal.
- We compute the derivative first to find where the function does nt change (hence it at its minimum)
  $-2/x^2 + 1 = 0$
- $x = \sqrt{2}$

- Plugging in back into z we get: $z = 2/\sqrt{2} + \sqrt{2} = 2.8284$
- Thus, the minimal sum of x and y is 2.82.
- Orinigally we had 3, thus she can spare an extra:
- $3 - 2.8284 = 0.1715$
- Since the price of both apples and seeds is 5 dollars, she can spend:
- $\Rightarrow \$5 * 0.1715 = \$0.857$

(b)

- We solve the same problem, we get:
- Original utility is: $U = 2^2 * 1 = 4$
- Setting up our equations we get: $x + y = z$, $x^2 y = 4$
- Equating the two gives: $z = 4/x^2 + x$
- Solving for the minimum gives:
  $4/x^2 + x = 0$
- $-8/x^3 + 1 = 0$
  $x = \sqrt[3]{8}$
- Substituting back in produces:
- $z = 4/(\sqrt[3]{8})^2 + \sqrt[3]{8} = 3$
- Thus, since the minimum z is 3, which is the amount we currently possess, leads us to conclude that Robert CANNOT spend any money on transportation and thus must remain in the forest!

[2]

(a)

(b)

- To find the efficient points in this situation, suppose we let man_2 start at any preassigned utility level, U_{m1}, then our problem become to choose X_1, Y_1, X_2, Y_2 to make man_1's utility as large as possible given man+1's utility constraint..
- Thus our lagrangian is as follows:
- $\ell = X_1 Y_1 + \lambda(X_2, Y_2^2 - U_{m1})$
- But we know that man 2 gets whatever man 1 doesn't, thus:
- $\ell = (280 - X_2)(280 - Y_2) + \lambda(X_2, Y_2^2 - U_{m1})$
- First order conditions are as follows:

  $\frac{\partial \ell}{\partial X_2} = (279)(280 - Y_2) + \lambda Y_2^2$
- $\frac{\partial \ell}{\partial Y_2} = (279)(280 - X_2) + \lambda 2 X_2 Y_2^2$

  $\frac{\partial \ell}{\partial \lambda} = X_2 Y_2^2 - U_{m1}$
- Given initial endowments of man 1 we compute his initial utility and then we show that he must trade in order to achieve maximum of both:
- From the above equations we compute Y:
- $Y = \frac{580 X_2}{280 + X_2}$
- $X = \frac{280 Y_2}{560 - Y_2}$

- Substituting into the last equation (with U_{m1}=) we get:
- $X_2(\frac{580X_2}{280+X_2})^2 - 3456000 = 0$
- Solving for X_2 with Matlab we get: X_2=117.5313
- And $(\frac{280Y_2}{560-Y_2})Y_2^2 - 3456000$ Y_2=168.9923
- Thus, we found that for man_2 168.9923 of Y and 117.5353 of X would produce the same utility.
- We know that total there is 280 of both X and Y.
- Thus for man_1 we have:
- X=280-117.5353 =111.47
- Y=280-168.9923=162.47
- Given these values, Man 1 will have a greater utility than previously, which proves that both man must trade to maximize utility!

[I didn't know how to find the price range]

(c)

- Now if we do not have first order conditions specified we cannot find a unique solution.
- From part (b) our first order conditions are:

  $\frac{\partial \ell}{\partial X_2} = (279)(280 - Y_2) + \lambda Y_2^2$
- $\frac{\partial \ell}{\partial Y_2} = (279)(280 - X_2) + \lambda 2 X_2 Y_2^2$

  $\frac{\partial \ell}{\partial \lambda} = X_2 Y_2^2 - U_{m1}$
- It can be clearly seen that if the initial U is not known, then we can maximize with any utility we choose…